# Review Set 2

This Review Set asks you to prepare written answers to questions on context-free grammars. Each of the questions has a short answer. You may discuss this Review Set with other students and work on the problems together.

1. Let $L_1$ be the language consisting of all non-empty palindromes over the alphabet $\Sigma = \{a, b\}$. That is, $L_1$ consists of all sequences of $a$'s and $b$'s that read the same forward or backward. For example, $aba \in L_1$ and and $bb \in L_1$ and $aabbbaa \in L_1$, but $abb \notin L_1$.

   Let $L_2$ be the language over $\Sigma = \{a, b\}$ denotated by the regular expression $a(a|b)*$.

   The language $L_3 = L_1 \cap L_2$ is context-free. A string $s$ is in $L_3$ if $s \in L_1$ and $s \in L_2$. Write a context-free grammar for the language $L_3$.

   **Optional Thing To Think About:** Is the intersection of a context-free language and a regular language always context-free?

2. Consider the following grammar:

$$
\begin{aligned}
S &\rightarrow aSb \\
S &\rightarrow Sb \\
S &\rightarrow \epsilon
\end{aligned}
$$

   (a) Give a one-sentence description of the language generated by this grammar.

   (b) Show that this grammar is ambiguous by giving a single string that can be parsed in two different ways. Draw both parse trees.

   (c) Give an unambiguous grammar that accepts the same language as the grammar above.

3. Using the context-free grammar for Cool given in the Cool Reference Manual, draw a parse tree for the following expression.

```
while not (x <- z <- 0) loop
  y <- z + 2 * x + 1
pool
```

Note that the context-free grammar by itself is ambiguous, so you will need to refer to the precedence and associativity rules to get the correct tree.

4. Give an example of a grammar that is $LL(3)$ but not $LL(2)$.