Earley Parsing and Examples





Outline

- Earley's Algorithm
 - Chart States
 - Operations
 - Example
- MyEarley.py
- PA3.jison
- Grammar "Conflicts"
 - Shift/Reduce



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Administrivia



- Midterm 1 will be next week
 - Likely: it will appear as a "Quiz" on Canvas
 - Pick any 90-minute slot on Thursday or Friday
 - Start any time. Once you start you have 90 minutes to finish.
 - Open notes, book, laptop, internet, "cool.exe", etc.
- Forbidden:
 - ChatGPT or similar services
 - I may "spot check" suspicious answers
 - Assistance from another live human
 - No posting new questions on StackOverflow

In One Slide

- Earley parsers are top-down and use dynamic programming. An Earley state records incremental information: when we started, what has been seen so far, and what we expect to see. The Earley chart holds a set of states for each input position. Shift, reduce and closure operations fill in the chart.
- You enjoy parsing. Parsing is approachable and fun.

Review: Earley States

- Let X be a non-terminal
- Let a and b be (possibly-empty) sequences of terminals and non-terminals
- Let $X \rightarrow ab$ be a production in your grammar
- Let j be a position in the input
- Each Earley State is a tuple $\langle X \rightarrow a \bullet b, j \rangle$
 - We are currently parsing an X
 - We have seen a, we expect to see b
 - We started parsing this *X* after seeing the first *j* tokens from the input.

Review: Earley Parse Table

- An Earley parsing table (or chart) is a onedimensional array. Each array element is a set of Earley states.
 - chart[i] holds the set of valid parsing states we could be in after seeing the first *i* input tokens
- Then the string tok₁...tok_n is in the language of a grammar with start symbol S iff
 - chart[n] contains < S → ab• , 0 > for some production rule S → ab in the grammar.
 - We then say the parser accepts the string.

Review: Filling In The Chart

- Three operations build up chart[n]
- The first is called shift or scan.
 - It corresponds to "seeing the next expected token" or "helping to confirm the current hypothesis" or "we're winning".

• Example:

- chart[1] contains $\langle E \rightarrow E \bullet + E, 0 \rangle$
- 2nd token is "+"
- Then put $\langle E \rightarrow E + \bullet E, 0 \rangle$ in chart[2]

Review: Filling In The Chart (2)

- The second operation is the closure or predictor.
 - It corresponds to "expanding rewrite rules" or "substituting in the definitions of non-terminals"
- Suppose the grammar is:

$$S \rightarrow E$$
 $E \rightarrow E + E \mid E - E \mid int$

• If chart[0] has $< S \rightarrow \bullet E$, 0 > then add

$$\langle E \rightarrow \bullet E + E, 0 \rangle$$

$$< E \rightarrow \bullet E - E , 0 >$$

$$< E \rightarrow \bullet int, 0 >$$

Review: Filling In The Chart (3)

- The third operation is reduction or completion.
 - It corresponds to "finishing a grammar rewrite rule" or "being done parsing a non-terminal" or "doing a rewrite rule in reverse and then shifting over the non-terminal".

Suppose:

```
- E \rightarrow int | E + E | E - E | (E), input is "(int"

- chart[2] contains < E \rightarrow int • , 1 >

- chart[1] contains < E \rightarrow (• E ), 0 >

- Then chart[2] += < E \rightarrow ( E • ), 0 >
```

Shift Practice

chart[3] contains

```
< S \rightarrow E \bullet , 0 > < E \rightarrow E \bullet - E , 0 > < E \rightarrow E \bullet - E , 0 > < E \rightarrow E \bullet - E , 2 > < E \rightarrow int \bullet , 2 >
```

• The 4th token is "+". What does shift bring in?

Shift Practice

chart[3] contains

```
< S \rightarrow E \bullet , 0 > < E \rightarrow E \bullet - E , 0 > < E \rightarrow E \bullet + E , 0 > < E \rightarrow E - E \bullet , 0 > < E \rightarrow E \bullet - E , 2 > < E \rightarrow int \bullet , 2 >
```

• The 4th token is "+". What does shift bring in?

$$< E \rightarrow E + \bullet E , 0>$$

 $< E \rightarrow E + \bullet E , 2 >$

... are both added to chart[4].

Closure Practice

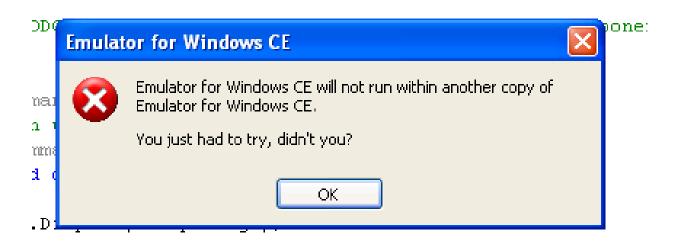
Grammar is

$$-S \rightarrow E$$
 $E \rightarrow E + E \mid E - E \mid (E) \mid int$

chart[4] contains:

$$\langle E \rightarrow E + \bullet E, 0 \rangle$$
 $\langle E \rightarrow E + \bullet E, 2 \rangle$

What does the closure operation bring in?



Closure Practice

Grammar is

$$-S \rightarrow E$$
 $E \rightarrow E + E \mid E - E \mid (E) \mid int$

chart[4] contains:

$$\langle E \rightarrow E + \bullet E, 0 \rangle$$
 $\langle E \rightarrow E + \bullet E, 2 \rangle$

What does the closure operation bring in?

$$<$$
 E \rightarrow • E + E , 4 $>$ $<$ E \rightarrow • E - E , 4 $>$ $<$ E \rightarrow • (E) , 4 $>$ $<$ E \rightarrow • int , 4 $>$... are all added to chart[4].

Reduction Practice

chart[4] contains:

```
< E \rightarrow E + \bullet E , 0 > < E \rightarrow E + \bullet E , 2 > < E \rightarrow \bullet E + E , 4 > < E \rightarrow \bullet E - E , 4 > < E \rightarrow \bullet (E), 4 > < E \rightarrow \bullet int, 4 >
```

chart[5] contains:

$$- < E \rightarrow int \bullet , 4 >$$

What does the reduce operator bring in?



Reduction Practice

chart[4] contains:

```
\langle E \rightarrow E + \bullet E, 0 \rangle \langle E \rightarrow E + \bullet E, 2 \rangle

\langle E \rightarrow \bullet E + E, 4 \rangle \langle E \rightarrow \bullet E - E, 4 \rangle

\langle E \rightarrow \bullet (E), 4 \rangle \langle E \rightarrow \bullet \text{ int }, 4 \rangle
```

chart[5] contains:

```
- < E \rightarrow int \bullet , 4 >
```

What does the reduce operator bring in?

$$\langle E \rightarrow E + E \bullet , 0 \rangle$$
 $\langle E \rightarrow E + E \bullet , 2 \rangle$ $\langle E \rightarrow E \bullet + E , 4 \rangle$

- ... are all added to chart[5]. (Plus more in a bit!)

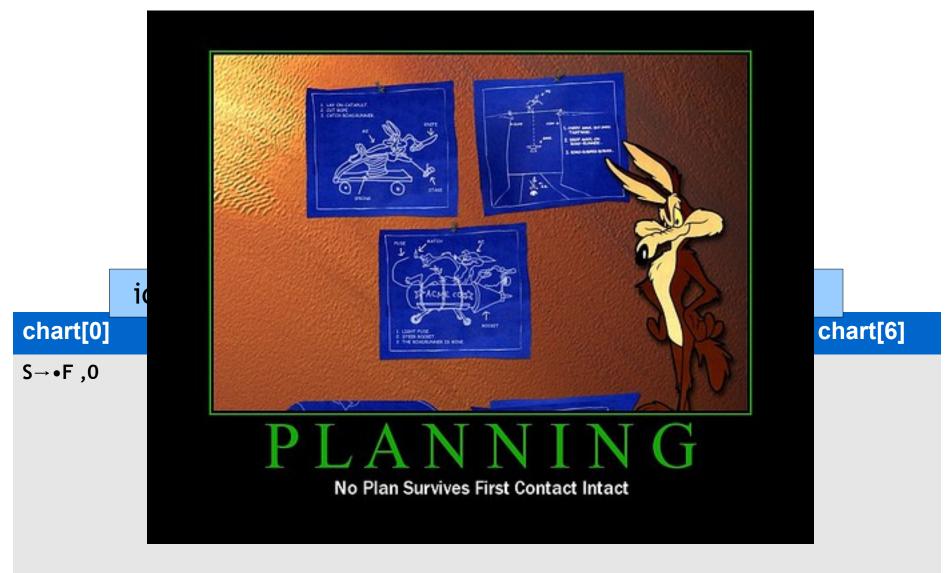
Earley Parsing Algorithm

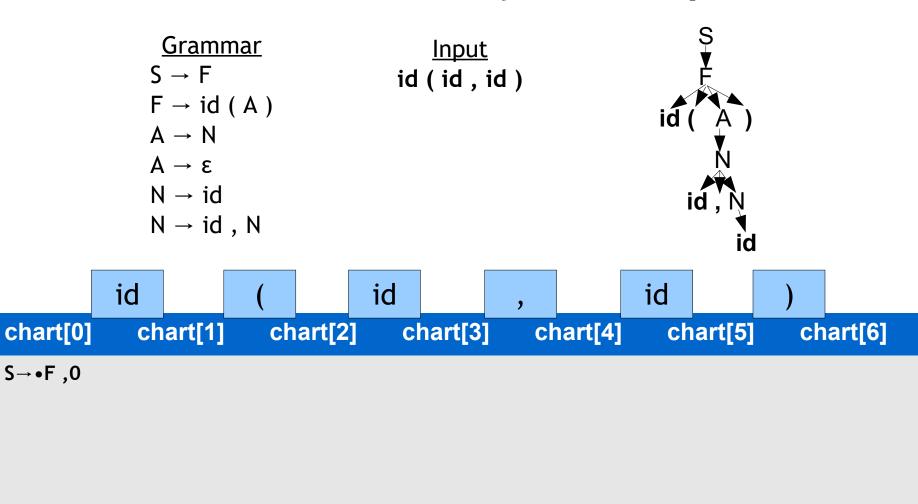
- Input: CFG G, Tokens tok₁...tok_n
- Work:

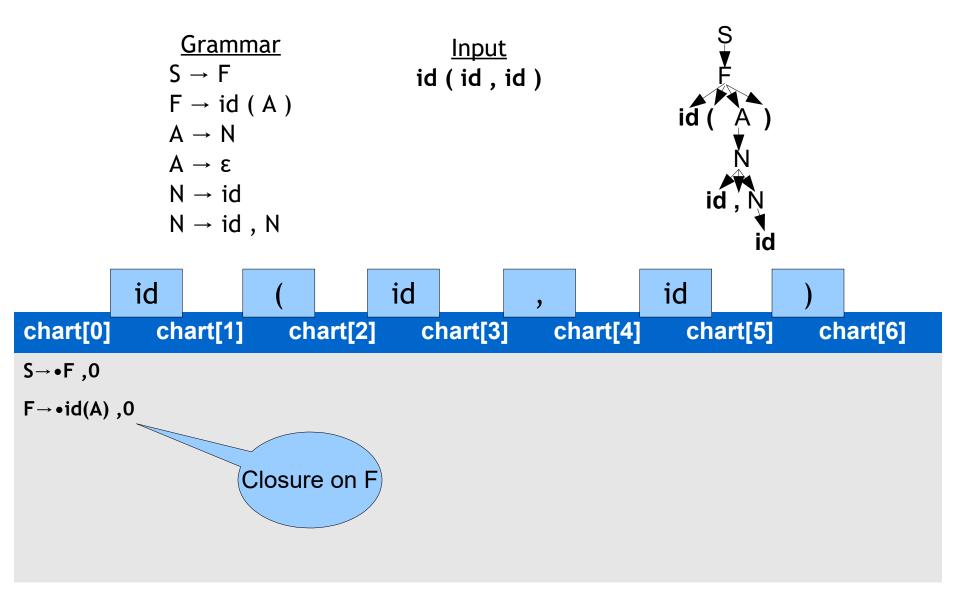
```
chart[0] = { < S → •ab , 0 > }
for i = 0 to n
  repeat
  use shift, reduce and closure on chart[i]
  until no new states are added
```

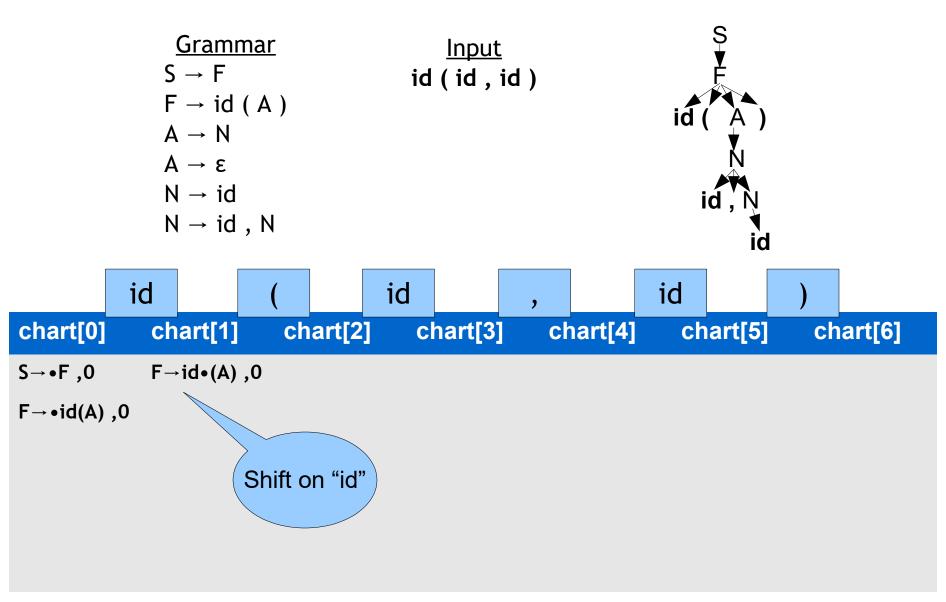
Output:

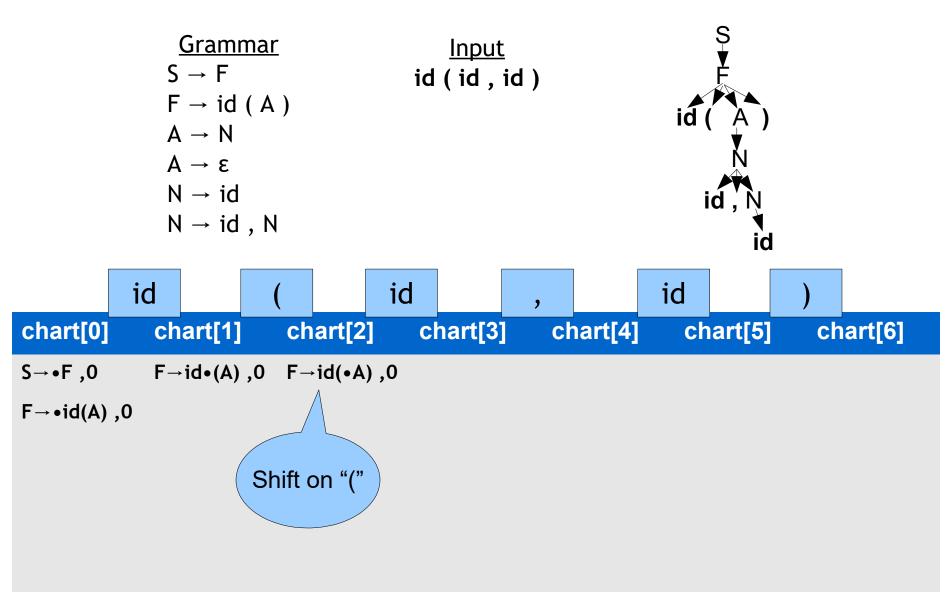
- true iff < S → ab• , 0 > in chart[n]

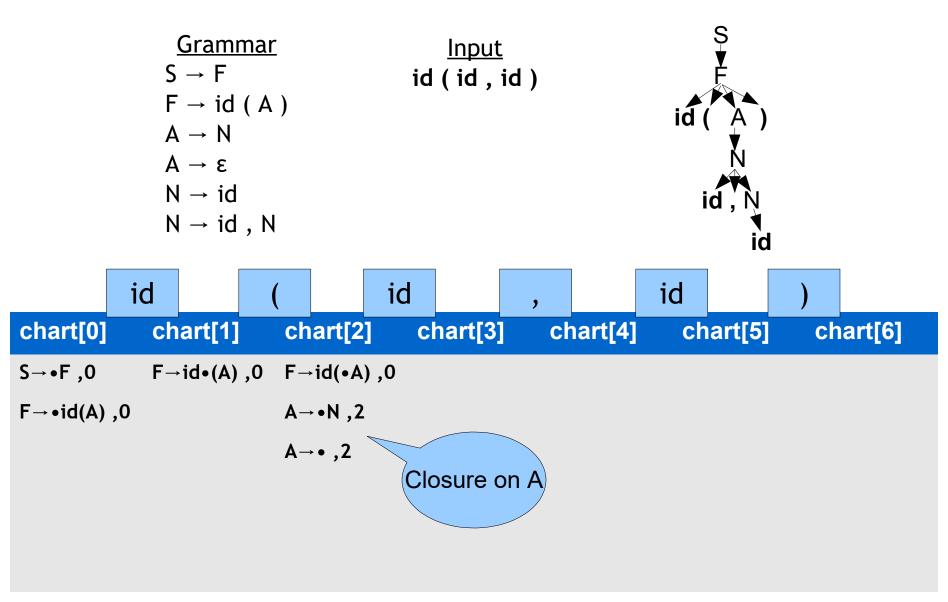


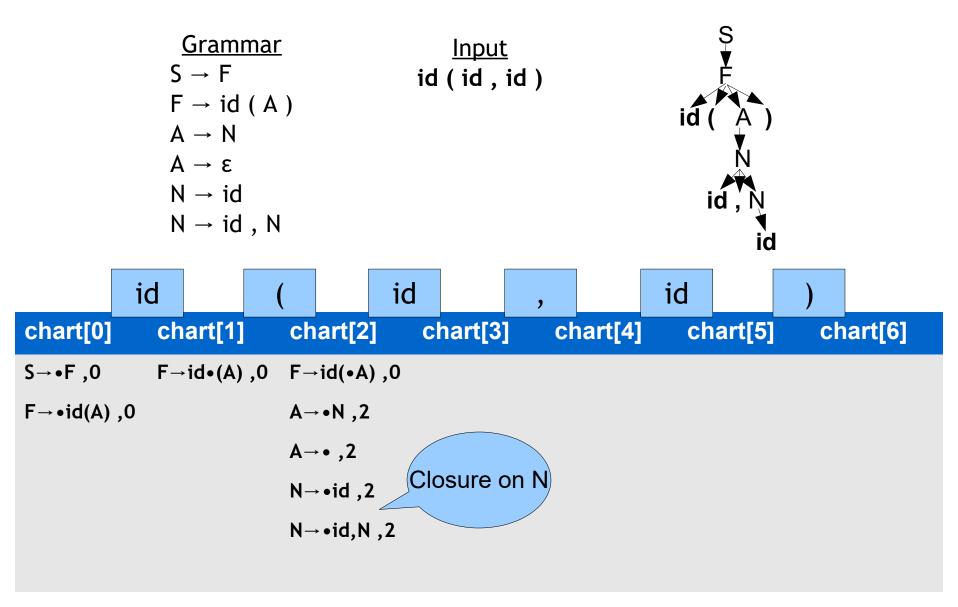


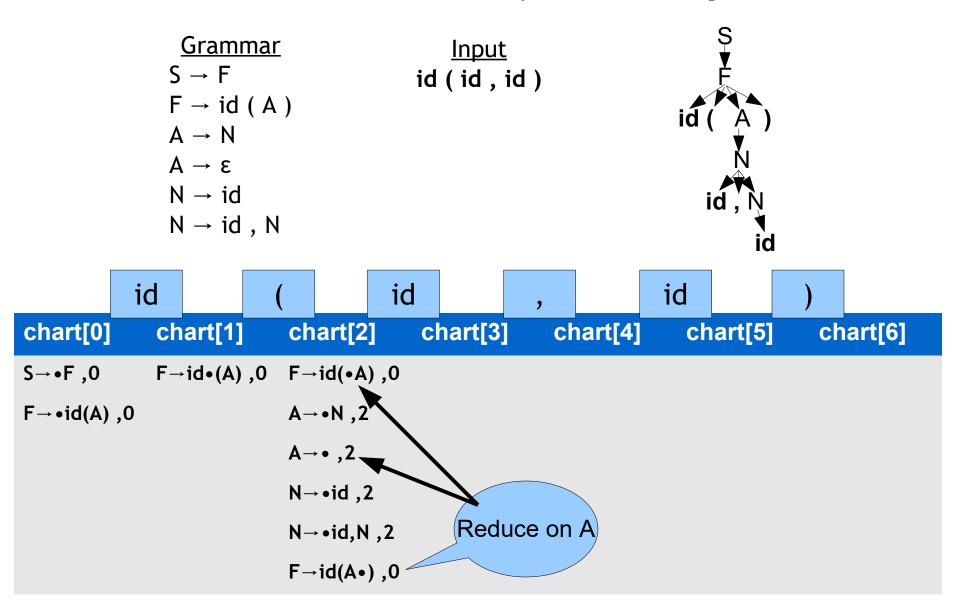


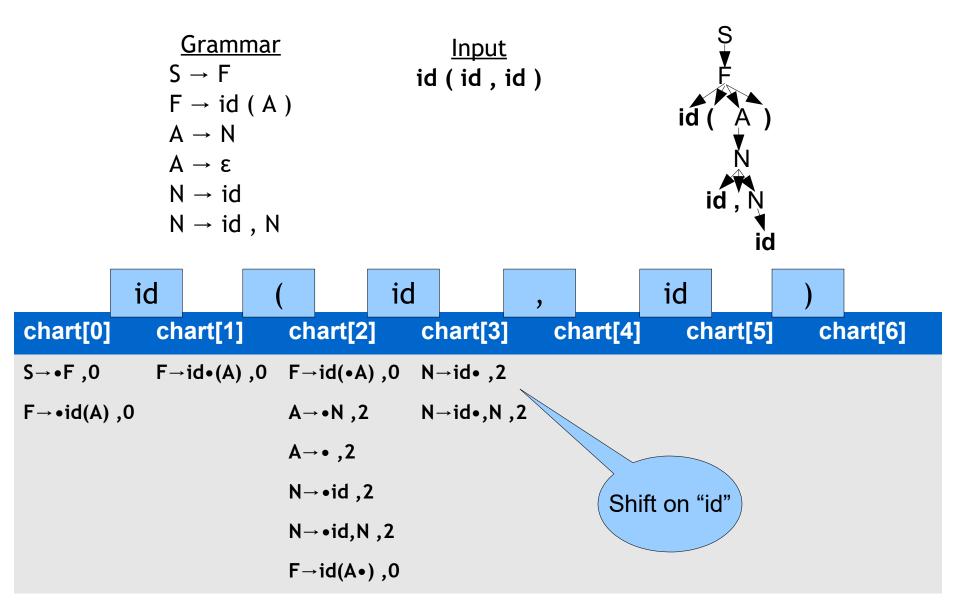


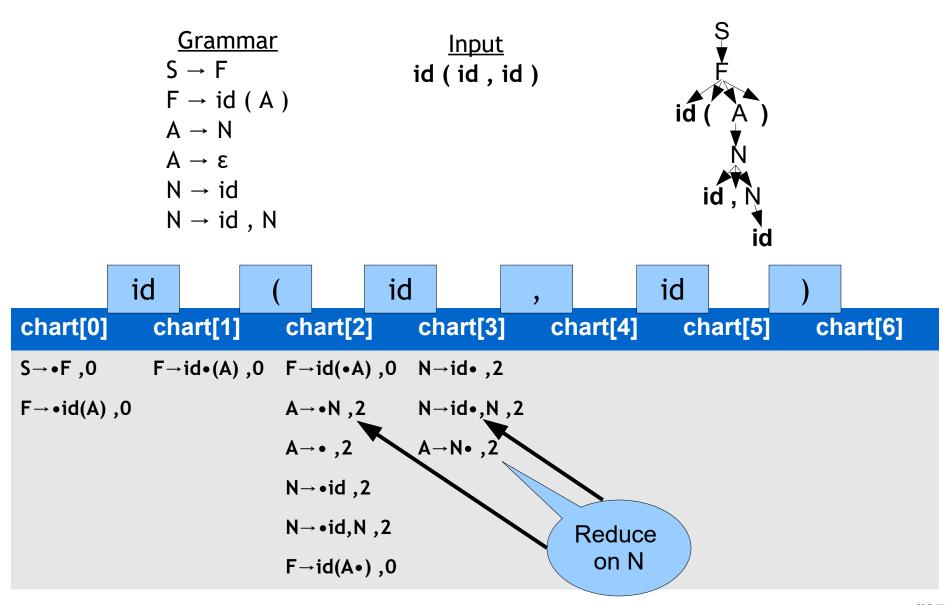


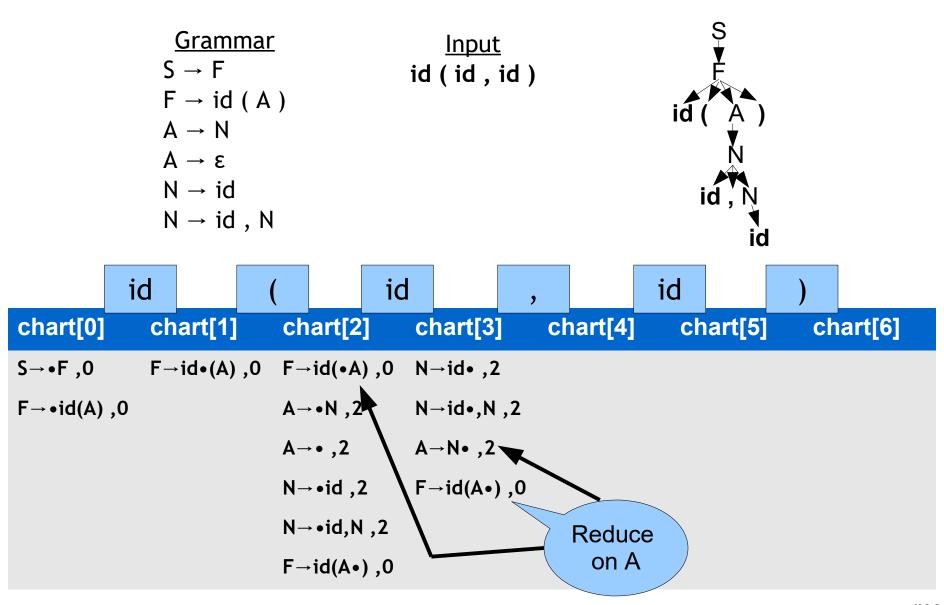


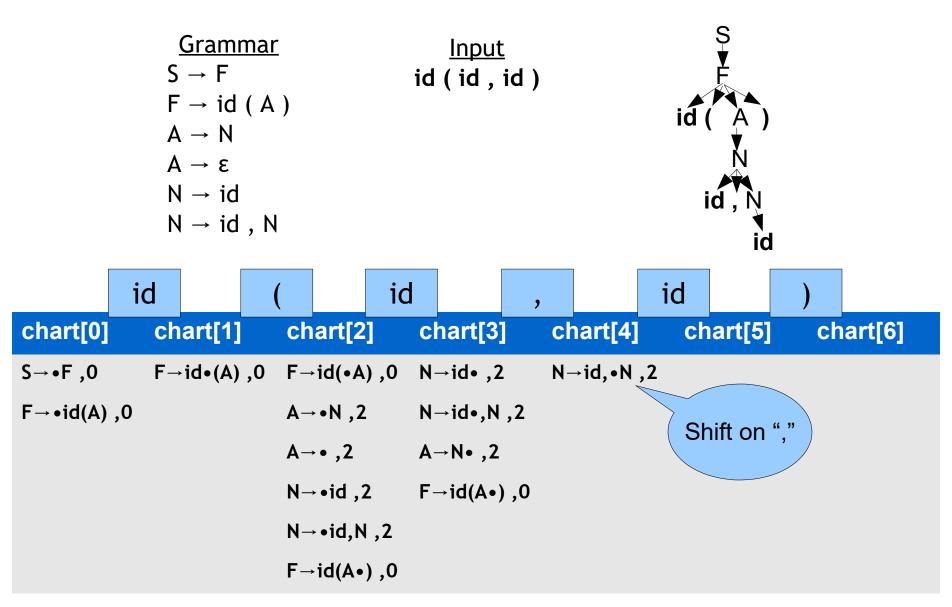


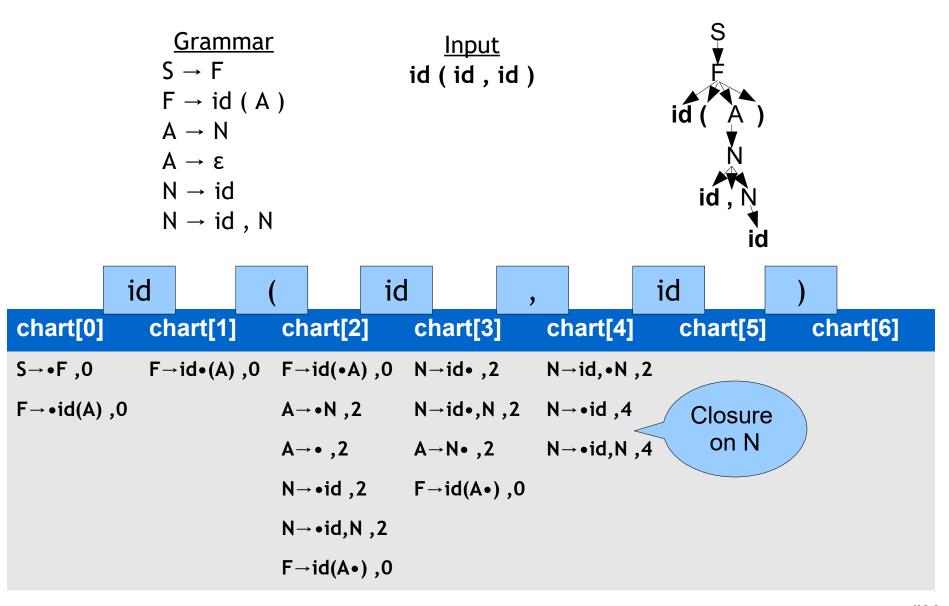


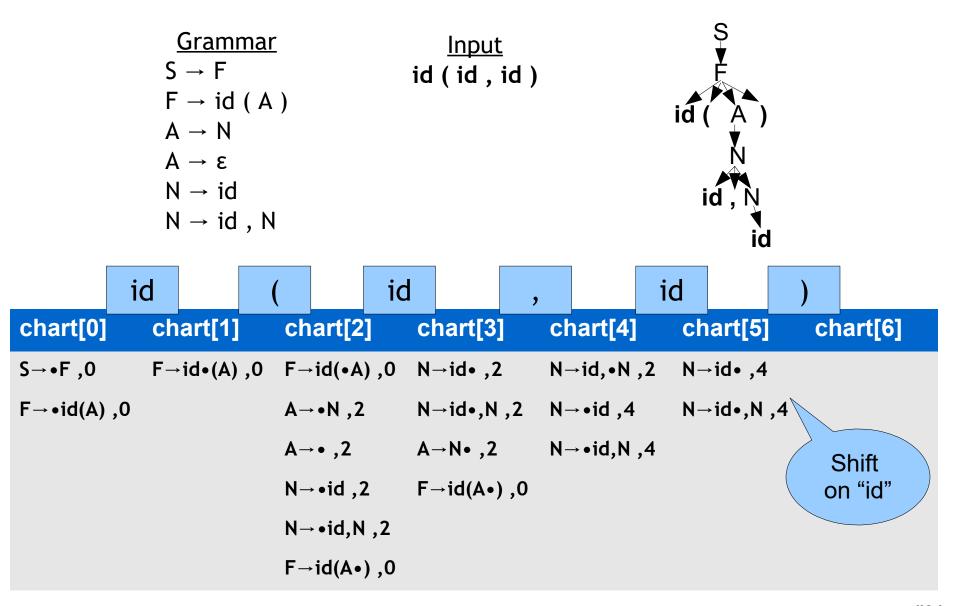


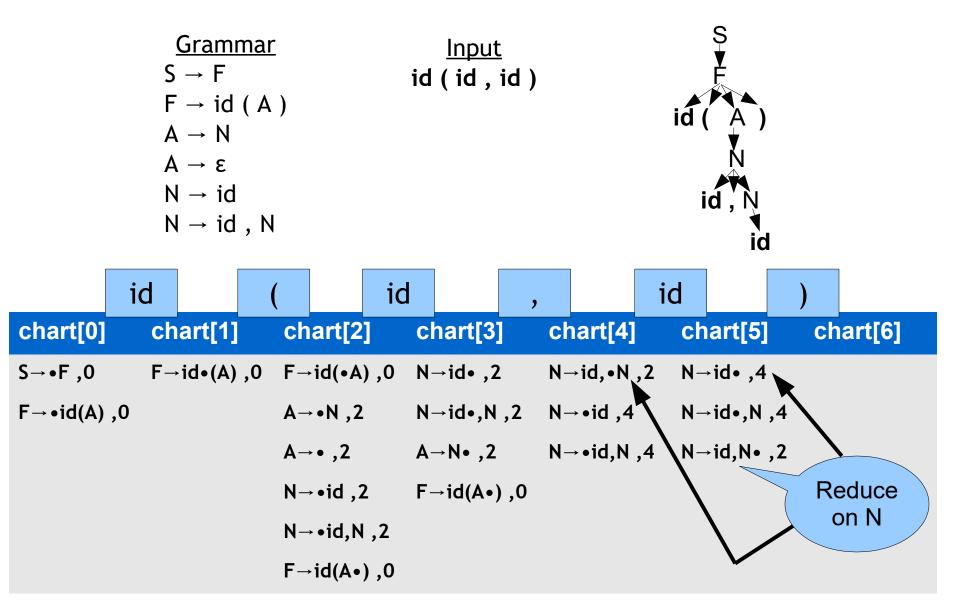


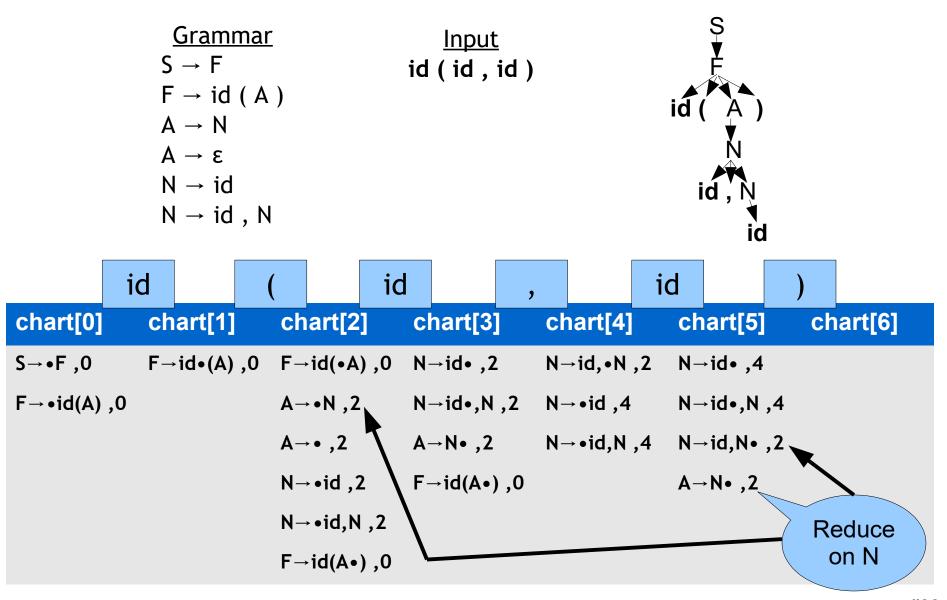


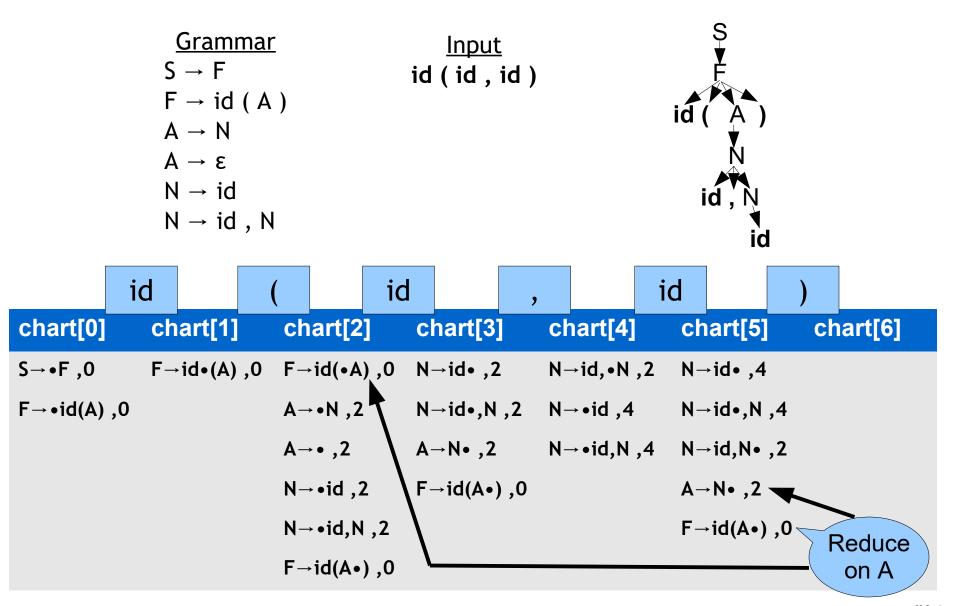


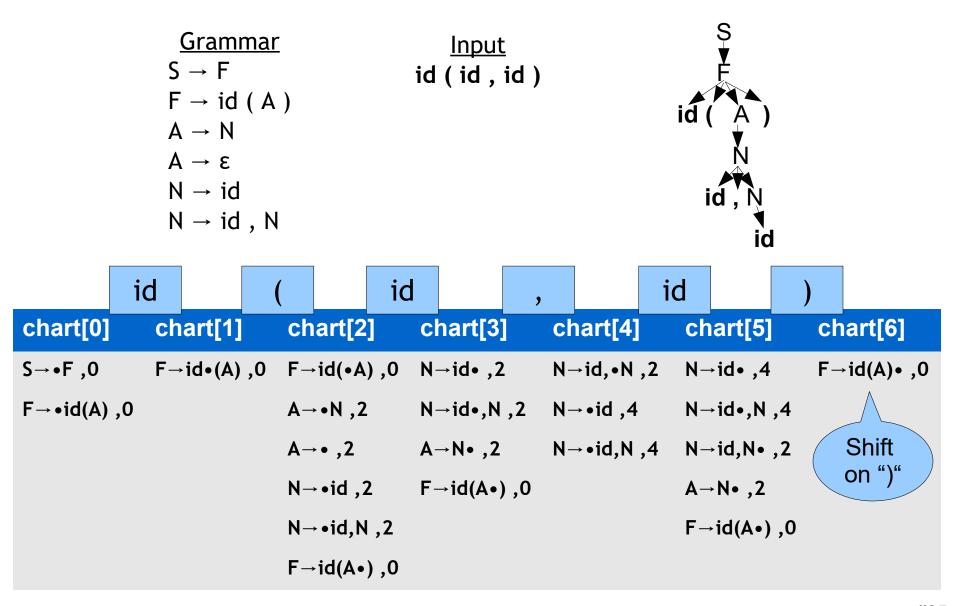


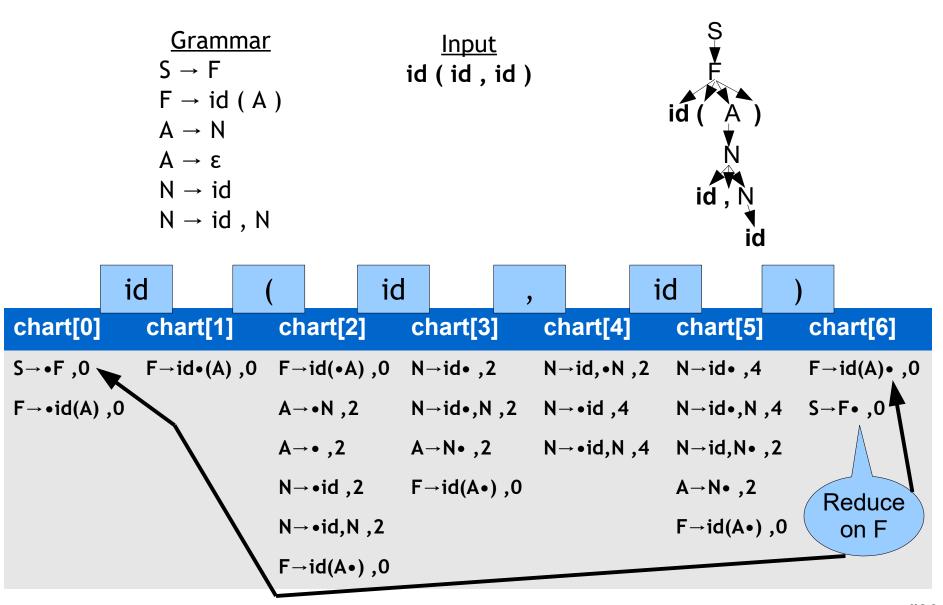




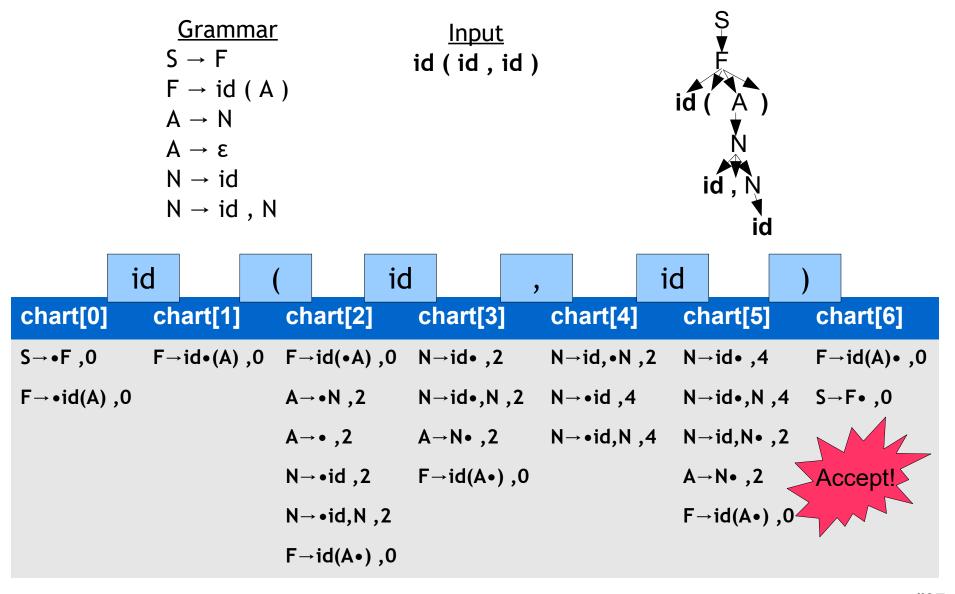








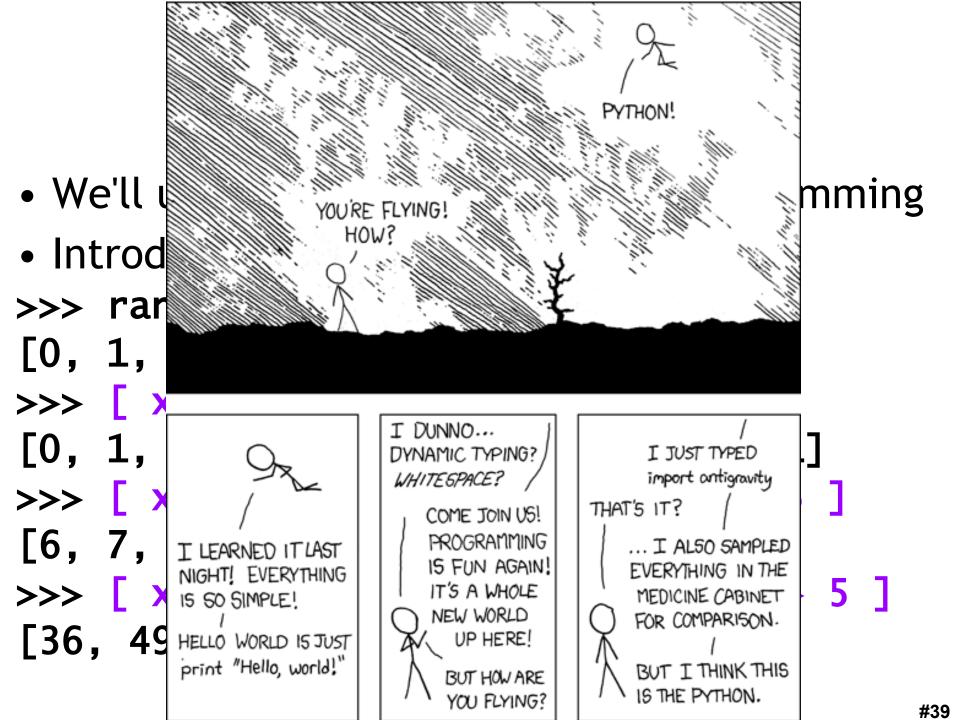
Massive Earley Example



Let's Implement It

- We'll use Python and Functional Programming
- Introducing List Comprehensions

```
>>> range(10)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> [ x*x for x in range(10) ]
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
>>> [x for x in range(10) if x > 5]
[6, 7, 8, 9]
>>> [ x*x for x in range(10) if x > 5 ]
[36, 49, 64, 81]
```



Data Structure Decisions

- For brevity, we'll use Lists and Tuples.
 - Not used: Named Tuples, Classes, etc.

```
grammar = [
  ("S", ["F"]),
  ("F", ["id", "(", "A", ")"]),
  ("A", [ ] ),
  ("A", ["N"]),
  ("N", ["id", ]),
  ("N", ["id", ",", "N"]),
tokens = [ "id" , "(" , "id", "," , "id", ")" ]
# X \rightarrow ab.cd, i == ("X",["a","b"],["c","d"],i)
```

Initialization

```
# By convention, the starting rule is
# the first rule in the grammar.
start_rule = grammar[0]
# The starting parse state is "S -> . abcd , from 0"
start_state = (start_rule[0], [], start_rule[1], 0)
# The parsing chart is a one-dimensional array,
# initially empty.
chart = {}
for i in range(len(tokens)+1): chart[i] = [ ]
# Start by placing the starting state in chart[0].
chart[0] = [ start_state ]
```

Shift

```
# If chart[i] contains "X -> ab.cd , from j"
# and c is token[i] then add:
# "X -> abc.d , from j" to chart[i+1]
def shift(tokens, i, x, ab, cd, j):
  if cd <> [] and tokens[i] == cd[0]:
    c = cd[0]
    d = cd[1:]
    abc = ab + [c]
    new\_chart\_state = (x, abc, d, j)
    new\_chart\_index = i + 1
    return [(new_chart_index, new_chart_state)]
  else:
    return |
```

Closure

```
# If chart[i] contains "X -> ab.cd , from j":
        and cd is not empty
        and c is a non-terminal
        and there is a grammar rule "c -> pqr"
# Then add:
        "c -> . pqr , from i"
        to chart[i]
def closure(grammar,i,x,ab,cd,j):
  return [ (i , (rule[0],[],rule[1],i)) \
        for rule in grammar \
        if cd <> [] and cd[0] == rule[0] ]
```

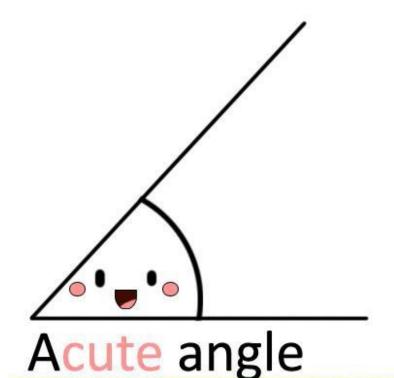
Reduction

```
# If chart[i] contains "X -> ab. , from j"
 (that is: cd is empty)
   and chart[j] contains "Y -> pq.Xr , from k"
# Then add
# "Y -> pqX.r , from k" to chart[i]
def reduction(chart,i,x,ab,cd,j):
  return [ (i, (jstate[0], jstate[1] + [x],
               (jstate[2])[1:], jstate[3] ))
    for jstate in chart[j]
    if cd == [] and jstate[2] <> []
                and (jstate[2])[0] == x]
```

Main Loop

```
# Step 2: Dynamic Programming
for i in range(len(tokens)):
  # Apply shift, closure and reduction until
  # no new parsing states are added to the chart.
  def apply_shift_closure_reduction():
    if any([add_to_chart(chart,
             shift(tokens,i,x,ab,cd,j) +
             closure(grammar,i,x,ab,cd,j) +
             reduction(chart,i,x,ab,cd,j))
            for x, ab, cd, j in chart[i] ]):
      apply_shift_closure_reduction()
      # do it again if any changes
  apply_shift_closure_reduction()
```

```
Example
grammar3 = [
 ("S", ["E"]),
 ("E", ["E", "-", "E"]),
 ("E", ["E", "+", "E"]),
 ("E", ["(", "E", ")"]),
 ("E", ["int"]),
tokens3 = [ "int", "-", "int" ]
chart[0]
 S \rightarrow E, from 0
                   , from 0
 E-> . E - E
 E \rightarrow E + E
                   . from 0
 E-> . (E)
                  , from 0
 E -> . int
                    , from 0
chart[1]
 E -> int .
                      , from 0
                      , from 0
 S \rightarrow E.
String Accepted: True
```



PA3 in JavaScript: parser.jison

```
%token PLUS MINUS INT
%left PLUS MINUS
%start program
%%
program: exp EOF { return $1; }
exp: exp PLUS exp { $$ = ["plus_node", $1, $3]; }
     exp MINUS exp { $$ = ["minus_node", $1,$3]; }
                   { $$ = ["int_node",
                           Number(yytext) ]; }
```

PA3 in JavaScript: main.js

```
var cl lex = [
  ['INT', "11"] ,
  'PLUS' ] .
  ['INT', "22"] ,
  ['MINUS'],
  ['INT', "33"],
  ['EOF'],
var token_count = 0
var parser =
    require("./parser").parser;
```

```
parser.lexer = {
  lex : function() {
   var cl_lex_entry =
        cl_lex[token_count++];
   var token = cl_lex_entry[0] ;
    var lexeme = cl_lex_entry[1] ;
    parser.lexer.yytext = lexeme ;
    return token;
  setInput : function(str) { }
var final_ast = parser.parse("");
console.log(final_ast);
```

PA3 in JavaScript Output:

PA3 Not Shown Here

- Reading in the .cl-lex file
- Handling line number information
- Printing out the AST in the desired format
- Adding parsing rules for whole classes and not just simple expressions
- Massive testing effort
 - diff vs. "cool --parse" requires "almost done"
- Dealing with ambiguity ("conflicts")
 - Let's do this one now.

Conflicts

- Add "%token NEG" and "exp: NEG exp".
- Oh noes:

```
Conflict in grammar: multiple actions possible when lookahead token is
PLUS in state 8
- reduce by rule: exp -> NEG exp
shift token (then go to state 6)
Conflict in grammar: multiple actions possible when lookahead token is
MINUS in state 8
- reduce by rule: exp -> NEG exp
- shift token (then go to state 7)
States with conflicts:
State 8
                           #lookaheads= EOF PLUS MINUS
  exp -> NEG exp .
  exp -> exp . PLUS exp
  exp -> exp . MINUS exp
```

Coi

- Add "%token NEG" a
- Oh noes:

Conflict in grammar: multiple acti PLUS in state 8

- reduce by rule: exp -> NEG exp
- shift token (then go to state 6

Conflict in grammar: multiple act MINUS in state 8

- reduce by rule: exp -> NEG exp
- shift token (then go to state 7)

States with conflicts:

```
State 8
 exp -> NEG exp .
 exp -> exp . PLUS exp
  exp -> exp . MINUS exp
```



Conflict Interpretation

- So some table entry has all three:
 - $exp \rightarrow NEG exp$.
 - $exp \rightarrow exp$. PLUS exp
 - $\exp \rightarrow \exp$. MINUS \exp
- What would the input have to look like to get to that table entry?



Internet Explorer

Question of the day: Which technological invention do you think has impacted our lives more - the telephone or the internet?

about a minute ago . Like . Comment



Conflict Interpretation

- So some table entry has all three:
 - $exp \rightarrow NEG exp$.
 - $exp \rightarrow exp$. PLUS exp
 - exp → exp . MINUS exp
- What would the input have to look like to get to that table entry?
 - NEG INT . PLUS INT

Conflict Interpretation

- So some table entry has all three:
 - $exp \rightarrow NEG exp$.
 - $exp \rightarrow exp$. PLUS exp
 - $exp \rightarrow exp$. MINUS exp
- What would the input have to look like to get

NEG

PLUS

to that table entry?

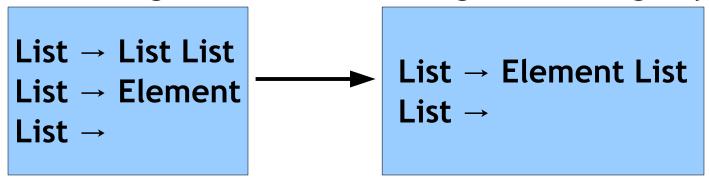
- NEG INT . PLUS INT

PLUS

NEG

Conflict Solution

- Shift/Reduce
 - Carefully specify precedence and associativity of operators (and sometimes of random tokens).
 - In last example, NEG has higher precedence than PLUS or MINUS.
- Reduce/Reduce
 - Rewrite grammar to avoid gross ambiguity:



Homework

- Midterm 1 Next Week
- PS3 recommended for next Tuesday
- PA3 due next Thursday