



Functional Programming

Introduction To Cool

Cunning Plan

- **ML Functional Programming**

- Fold
- Sorting

- **Cool Overview**

- Syntax
- Objects
- Methods
- Types



One-Slide Summary

- In **functional programming**, functions are first-class citizens that operate on, and produce, immutable data.
- Functions and type inference are **polymorphic** and operate on more than one type (e.g., `List.length` works on int lists and string lists).
- Ocaml and Haskell (and Cool) support **pattern matching** over user-defined data types.
- **fold** is a powerful and general higher-order function. It can simulate many others.
- **Cool** is an object-oriented language with enough features to be indicative of modern practice.

Pattern Matching (Error?)

- Simplifies Code (eliminates ifs, accessors)

```
type btree = (* binary tree of strings *)
```

```
| Node of btree * string * btree
```

```
| Leaf of string
```

```
let rec height tree = match tree with
```

```
| Leaf _ -> 1
```

```
| Node(x,_,y) -> 1 + max (height x) (height y)
```

```
let rec mem tree elt = match tree with
```

```
| Leaf str | Node(_,str,_) -> str = elt
```

```
| Node(x,_,y) -> mem x elt || mem y elt
```

Pattern Matching (Error?)

- Simplifies Code (eliminates ifs, accessors)

```
type btree = (* binary tree of strings *)
```

```
| Node of btree * string * btree
```

```
| Leaf of string
```

```
let rec height tree = match tree with
```

```
| Leaf _ -> 1
```

```
| Node(x,_,y) -> 1 + max (height x) (height y)
```

```
let rec mem tree elt = match tree with
```

```
| Leaf str | Node(_,str,_) -> str = elt
```

```
| Node(x,_,y) -> mem x elt || mem y elt
```

bug?

Pattern Matching (Error!)

- Simplifies Code (eliminates ifs, accessors)

```
type btree = (* binary tree of strings *)
```

```
| Node of btree * string * btree
```

```
| Leaf of string
```

```
let rec bad tree elt = match tree with
```

```
| Leaf str | Node(_,str,_) -> str = elt
```

```
| Node(x,_,y) -> bad x elt || bad y elt
```

```
let rec mem tree elt = match tree with
```

```
| Leaf str | Node(_,str,_) when str = elt -> true
```

```
| Node(x,_,y) -> mem x elt || mem y elt
```

Recall: Polymorphism

- Functions and type inference are polymorphic

- Operate on more than one type

- let rec length x = match x with

- | [] -> 0

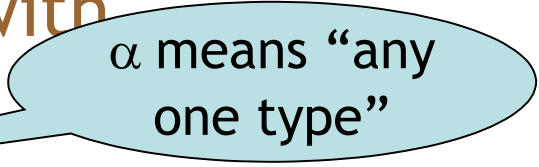
- | hd :: tl -> 1 + length tl

- val length : α list -> int

- length [1;2;3] = 3

- length ["algol"; "smalltalk"; "ml"] = 3

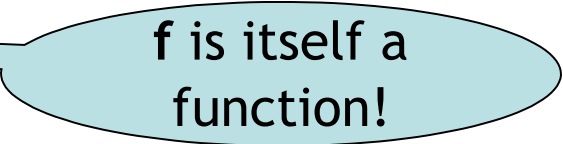
- length [1 ; "algol"] = type error!



α means “any one type”

Recall: Higher-Order Functions

- Function are first-class values
 - Can be used whenever a value is expected
 - Notably, can be passed around
 - Closure captures the environment
 - **let rec map f lst = match lst with**
 - **| [] -> []**
 - **| hd :: tl -> f hd :: map f tl**
 - **val map : (α -> β) -> α list -> β list**
 - **let offset = 10 in**
 - **let myfun x = x + offset in**
 - **val myfun : int -> int**
 - **map myfun [1;8;22] = [11;18;32]**
- Extremely powerful programming technique
 - General iterators
 - Implement abstraction



f is itself a function!

Fold

- The fold operator comes from Recursion Theory (Kleene, 1952)

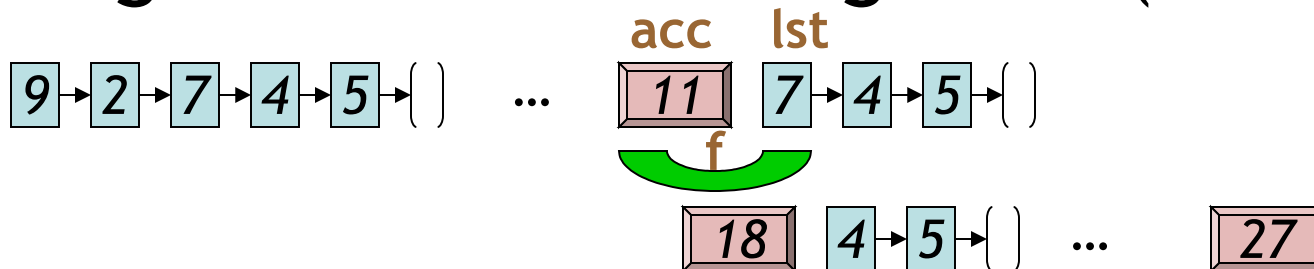
let rec fold f acc lst = match lst with

| [] -> acc

| hd :: tl -> fold f (f acc hd) tl

- val fold : ($\alpha \rightarrow \beta \rightarrow \alpha$) -> $\alpha \rightarrow \beta$ list -> α

- Imagine we're summing a list (f = addition):



It's Lego Time

- Let's build things out of Fold!
 - **length** lst = fold (fun acc elt -> acc + 1) 0 lst
 - **sum** lst = fold (fun acc elt -> acc + elt) 0 lst
 - **product** lst=fold (fun acc elt -> acc * elt) 1 lst
 - **and** lst = fold (fun acc elt -> acc & elt) true lst
- How would we do **or**?
- How would we do **reverse**?



Referential Transparency

- To find the meaning of a functional program we replace each reference to a variable with its definition.
 - This is called **referential transparency**.

- Example:

let $y = 55$

let $f\ x = x + y$

$f\ 3$

--> means --> $3 + y$

--> means --> $3 + 55$

Worked Example: Product

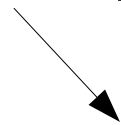
```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
fold (*) 1 [8;6;7]
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 1 [8;6;7]

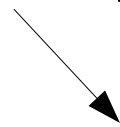


```
match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 1 [8;6;7]



with f=*, acc=1, and lst=[8;6;7]

```
match lst with
```

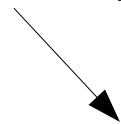
```
| [] -> acc
```

```
| hd :: tl -> fold f (f acc hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 1 [8;6;7]



```
match [8;6;7] with  
| [] -> 1  
| hd :: tl -> fold (*) (* 1 hd) tl
```

Worked Example: Product


```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
match [8;6;7] with  
| [] -> 1  
| hd :: tl -> fold (*) (* 1 hd) tl
```


Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [8;6;7] in  
fold (*) (* 1 hd) tl
```



```
match [8;6;7] with  
| [] -> 1  
| hd :: tl -> fold (*) (* 1 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [8;6;7] in  
fold (*) (* 1 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [8;6;7] in  
fold (*) (* 1 hd) tl
```

←

```
fold (*) (* 1 8) [6;7]
```

Worked Example: Product

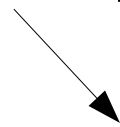
```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
fold (*) 8 [6;7]
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 8 [6;7]



with f=*, acc=8, and lst=[6;7]

```
match lst with
```

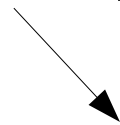
```
| [] -> acc
```

```
| hd :: tl -> fold f (f acc hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 8 [6;7]



```
match [6;7] with
```

```
| [] -> 8
```

```
| hd :: tl -> fold (*) (* 8 hd) tl
```

Worked Example: Product


```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
match [6;7] with  
| [] -> 8  
| hd :: tl -> fold (*) (* 8 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [6;7] in  
fold (*) (* 8 hd) tl
```



```
match [6;7] with  
| [] -> 8  
| hd :: tl -> fold (*) (* 8 hd) tl
```


Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [6;7] in  
fold (*) (* 8 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [6;7] in  
fold (*) (* 8 hd) tl
```

←

```
fold (*) (* 8 6) [7]
```

Worked Example: Product

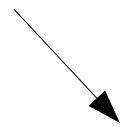
```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
fold (*) 48 [7]
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 48 [7]



with f=*, acc=48, and lst=[7]

```
match lst with
```

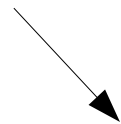
```
| [] -> acc
```

```
| hd :: tl -> fold f (f acc hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 48 [7]



match [7] with

| [] -> 48

| hd :: tl -> fold (*) (* 48 hd) tl

Worked Example: Product


```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
match [7] with  
| [] -> 48  
| hd :: tl -> fold (*) (* 48 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [7] in  
fold (*) (* 48 hd) tl
```



```
match [7] with  
| [] -> 48  
| hd :: tl -> fold (*) (* 48 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
let hd :: tl = [7] in  
fold (*) (* 48 hd) tl
```


Worked Example: Product

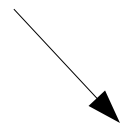
```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
fold (*) (* 48 7) []
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

fold (*) 336 []



with f=*, acc=336, and lst=[]

```
match lst with
```

```
| [] -> acc
```

```
| hd :: tl -> fold f (f acc hd) tl
```

Worked Example: Product

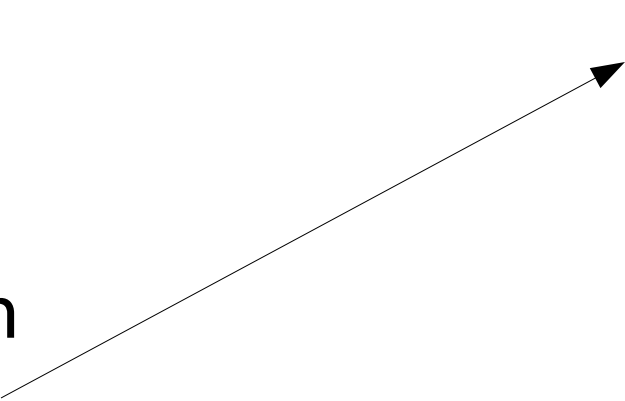
```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

```
match [] with  
| [] -> 336  
| hd :: tl -> fold (*) (* 336 hd) tl
```

Worked Example: Product

```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```

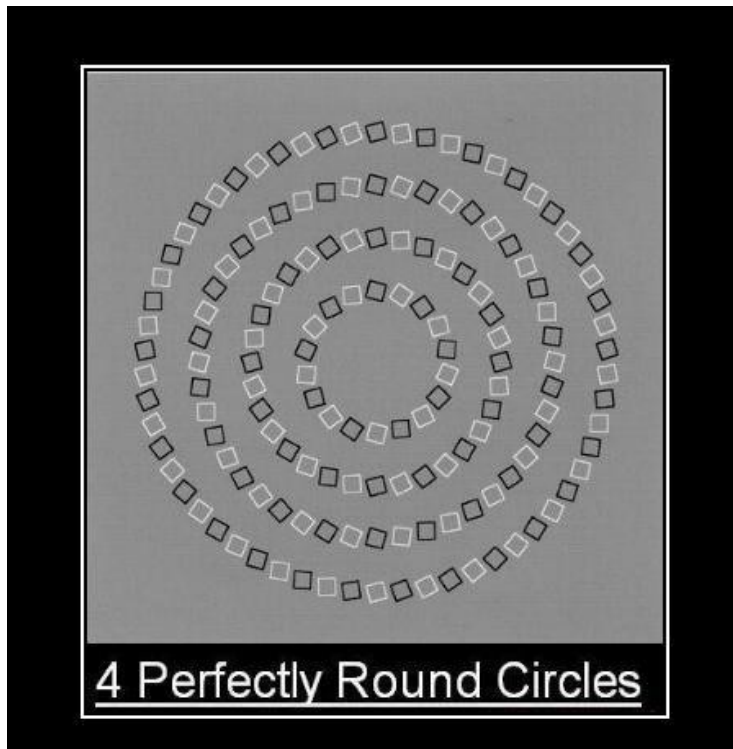
```
match [] with  
| [] -> 336  
| hd :: tl -> fold (*) (* 336 hd) tl
```



336

Worked Example: Product

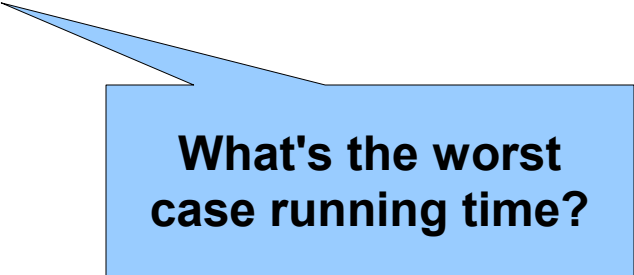
```
let rec fold f acc lst = match lst with  
| [] -> acc  
| hd :: tl -> fold f (f acc hd) tl
```



336

Insertion Sort in OCaml

```
let rec insert_sort cmp lst =  
  match lst with  
  | [] -> []  
  | hd :: tl -> insert cmp hd (insert_sort cmp tl)  
and insert cmp elt lst =  
  match lst with  
  | [] -> [elt]  
  | hd :: tl when cmp hd elt ->  
    hd :: (insert cmp elt tl)  
  | _ -> elt :: lst
```



What's the worst case running time?

Sorting Examples

- `langs = ["fortran"; "algol"; "c"]`
- `courses = [216; 333; 415]`
- `sort (fun a b -> a < b) langs`
 - ["algol"; "c"; "fortran"]
- `sort (fun a b -> a > b) langs`
 - ["fortran"; "c"; "algol"]
- `sort (fun a b -> strlen a < strlen b) langs`
 - ["c"; "algol"; "fortran"]
- `sort (fun a b -> match is_odd a, is_odd b with
| true, false -> true (* odd numbers first *)
| false, true -> false (* even numbers last *)
| _, _ -> a < b (* otherwise ascending *)) courses`
 - [333 ; 415 ; 216]

*Java uses
Inner Classes
for this.*

Broadly Available

- **ML, Python and Ruby** all support functional programming
 - closures, anonymous functions, etc.
- ML has strong static typing and type inference (as in this lecture)
- Ruby and Python have “strong” dynamic typing (or duck typing)
- All three combine OO and Functional
 - ... although it is rare to use both.

Modern Languages

- This is the most widely-spoken first language in the European Union. It is the third-most taught foreign language in the English-speaking world, after French and Spanish. Its word order is a bit more relaxed than English (since nouns are inflected to indicate their cases, as in Latin) - infamously, verbs often appear at the very end of a subordinate clause. The language's famous “Storm and Stress” movement produced classics such as *Faust*.

Natural Languages

- This linguist and cognitive scientist is famous for, among other things, the sentence **“Colorless green ideas sleep furiously”**. Introduced in his 1957 work *Syntactic Structures*, the sentence is correct but has not understandable meaning, thus demonstrating the distinction between syntax and semantics. Compare **“Time flies like an arrow; fruit flies like a banana.”** which illustrates garden path syntactic ambiguity.

Cool Overview

- Classroom Object-Oriented Language
- Designed to
 - Be implementable in one semester
 - Give a taste of implementing modern features
 - Abstraction
 - Static Typing
 - Inheritance
 - Dynamic Dispatch
 - And more ...
 - But many “grungy” things are left out

A Simple Example

```
class Point {  
    x : Int <- 0;  
    y : Int <- 0;  
};
```

- Cool programs are sets of class definitions
 - A special **Main** class with a special method **main**
 - Classes are like those in Java or Python or C++
- class = a collection of fields and methods
- Instances of a class are objects

Cool Objects

```
class Point {  
    x : Int <- 0;  
    y : Int; (* use default value *)  
};
```

- The expression “**new Point**” creates a new object of class **Point**
- An object can be thought of as a record with a slot for each attribute (= field)

x	y
0	0

Methods

```
class Point {  
  x : Int <- 0;  
  y : Int <- 0;  
  movePoint(newx : Int, newy : Int) : Point {  
    { x <- newx;  
      y <- newy;  
      self;  
    } -- close block expression  
  }; -- close method  
}; -- close class
```

- A class can also define methods for manipulating its attributes
- Methods refer to the current object using **self**

Aside: Semicolons

```
class Point {  
  x : Int <- 0;  
  y : Int <- 0;  
  movePoint(newx : Int, newy : Int) : Point {  
    { x <- newx;  
      y <- newy;  
      self;  
    } -- close block expression  
  }; -- close method  
}; -- close class
```

Yes, it's
somewhat arbitrary.
Still, don't get it wrong.



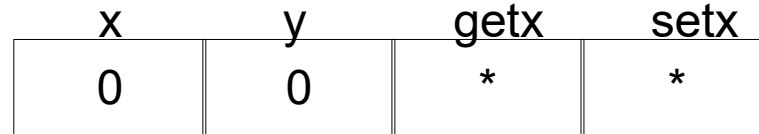
Information Hiding

- Methods are **global**
- Attributes are **local** to a class
 - They can *only* be accessed by *that class's methods*

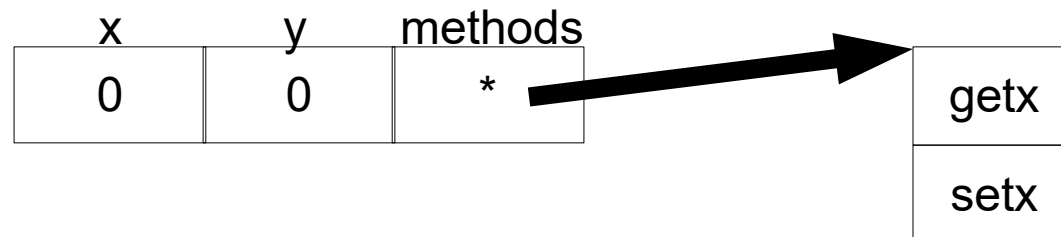
```
class Point {  
  x : Int <- 0;  
  y : Int <- 0;  
  getx () : Int { x } ;  
  setx (newx : Int) : Int { x <- newx };  
};
```


Methods and Object Layout

- Each object knows how to access the code of its methods
- As if the object contains a slot pointing to the code



- In reality, implementations save space by sharing these pointers among instances of the same class



Inheritance

- We can extend points to color points using **subclassing** => **class hierarchy**

```
class ColorPoint extends Point {  
  color : Int <- 0;  
  movePoint(newx:Int, newy:Int) : Point {  
    { color <- 0;  
      x <- newx; y <- newy;  
      self;  
    }  
  };  
};
```

Note references to fields x y –
They're defined in Point!

x	y	color	movePoint
0	0	0	*

Kool Types

- Every class is a **type**
- Base (built-in, predefined) classes:
 - **Int** for integers
 - **Bool** for booleans: **true**, **false**
 - **String** for strings
 - **Object** root of class hierarchy
- All variables must be declared
 - compiler infers types for expressions (like Java)



Cool Type Checking

- **x : Point;**
- **x <- new ColorPoint;**
- ... is well-typed if **Point** is an ancestor of **ColorPoint** in the class hierarchy
 - Anywhere a **Point** is expected, a **ColorPoint** can be used (Liskov, ...)
- Rephrase: ... is well-typed if **ColorPoint** is a subtype of **Point**
- Type safety: a well-typed program *cannot* result in run-time type errors

Method Invocation and Inheritance

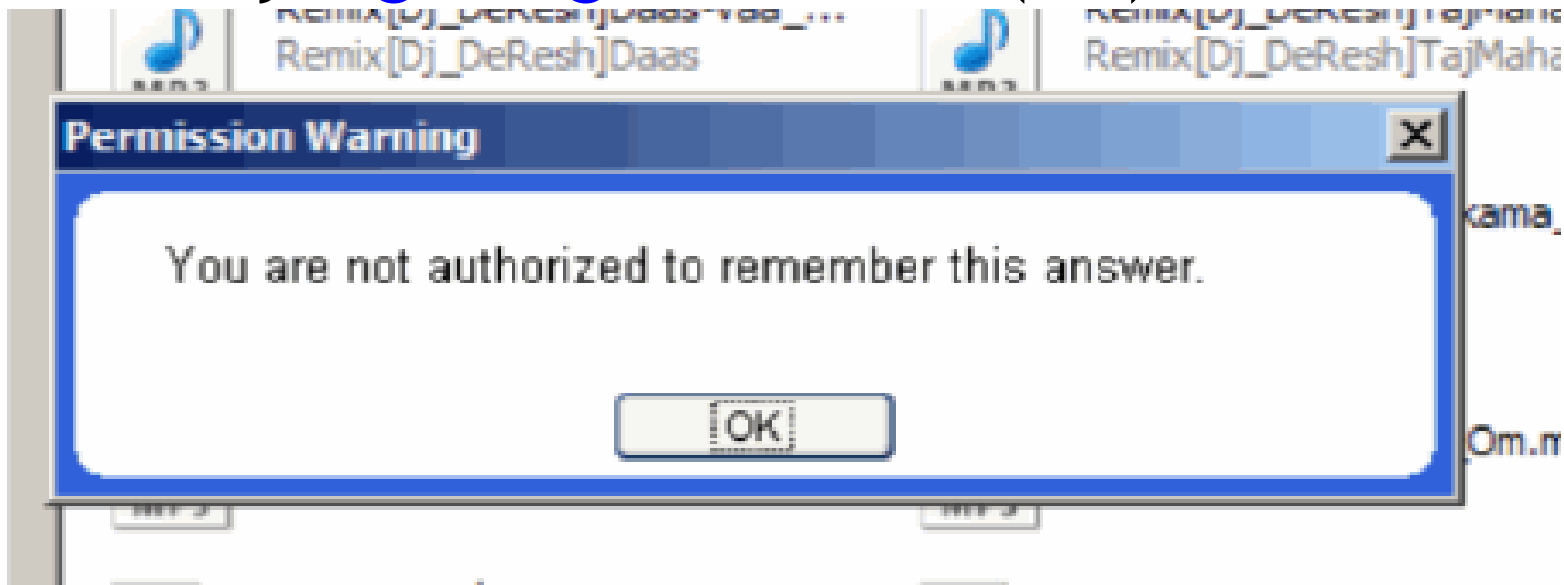
- Methods are invoked by (dynamic) dispatch
- Understanding dispatch in the presence of inheritance is a subtle aspect of OO
 - `p : Point;`
 - `p <- new ColorPoint;`
 - `p.movePoint(1,2);`
- `p` has static type `Point`
- `p` has dynamic type `ColorPoint`
- `p.movePoint` must invoke `ColorPoint` version

Other Expressions

- Cool is an expression language (like Ocaml)
 - Every expression has a type and a value
 - Conditionals `if E then E else E fi`
 - Loops `while E loop E pool`
 - Case/Switch `case E of x : Type => E ; ... esac`
 - Assignment `x <- E`
 - Primitive I/O `out_string(E), in_string(), ...`
 - Arithmetic, Logic Operations, ...
- Missing: arrays, floats, interfaces, exceptions
 - Plus: you tell me!

Cool Memory Management

- Memory is allocated every time “**new E**” executes
- Memory is deallocated automatically when an object is not reachable anymore
 - Done by a **garbage collector** (GC)



Course Project

- A complete **interpreter**
 - Cool Source ==> Executed Program
 - No optimizations
 - Also no GC
- Split in 4 programming assignments (PAs)
- There is adequate time to complete assignments
 - But start early and follow directions
- PA2-5 ==> individual or teams (of max 2)

Real-Time OCaml Demo

- I will code up these, with explanations, until time runs out.
 - Read in a list of integers and print the sum of all of the odd inputs.
 - Read in a list of integers and determine if any sublist of that input sums to zero.
 - Read in a directed graph and determine if node END is reachable from node START.
- You pick the order.
- Bonus: Asymptotic running times?

Homework

- PA1 Checkpoint
- Reading: Chapters 2.1 - 2.2, On-Line
- Bonus for getting this far: questions about fold are very popular on tests! If I say “write me a function that does foozle to a list”, you should be able to code it up with fold.